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Determination of π^0 meson quadrupole polarizabilities from the process $\gamma\gamma \to \pi^0\pi^0$

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A fit of the experimental data to the total cross section of the process $\gamma\gamma \to \pi^0\pi^0$ in the energy region from 270 to 2250 MeV has been carried out using dispersion relations for the invariant amplitudes where the quadrupole polarizabilities are free parameters. As a result the sum and difference of the electric and magnetic quadrupole polarizabilities of the π^0 meson have been found for the first time: $(\alpha_2+\beta_2)_{\pi^0} = (-0.181 \pm 0.004) \times 10^{-4}\,\mathrm{fm}^5$, $(\alpha_2-\beta_2)_{\pi^0} = (39.70 \pm 0.02) \times 10^{-4}\,\mathrm{fm}^5$. In addition, dispersion sum rules have been constructed for this sum and difference, respectively. The values of $(\alpha_2-\beta_2)_{\pi^0}$ and $(\alpha_2+\beta_2)_{\pi^0}$ extracted from the experimental data on the process $\gamma\gamma \to \pi^0\pi^0$ are in good agreement with the result of calculations in the framework of these dispersion sum rules.

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I. INTRODUCTION

Hadron polarizabilities are structure parameters, the values of which are very sensitive to predictions of different theoretical models. Therefore, accurate experimental determination of these values provides a method for testing the validity of such models. However, because there are no pion targets, it is necessary to use indirect methods to determine the pion polarizabilities. For example, the dipole polarizabilities of charged pions can be determined either from the scattering of the high energy pions off the Coulomb field of heavy nuclei [1, 2, 3] or from radiative π^+ photoproduction from the proton [4, 5, 6]. Unfortunately, the results obtained from analyses of the reaction $\gamma\gamma \to \pi\pi$ at low energies [7, 8, 9] are essentially model dependent due to the strong S-wave $\pi\pi$ interaction in this energy region.

Moreover, due to the fact that the Born term for the reaction $\gamma\pi^0 \to \gamma\pi^0$ is equal to zero, extraction of the π^0 meson polarizabilities by extrapolating the experimental data on the radiative π^0 photoproduction from the proton to the pion pole is ineffective. At present the most reliable method in this case is an analysis of the process $\gamma\gamma \to \pi^0\pi^0$ in the region of the $f_2(1270)$ meson where the cross section of this process is very sensitive to the values of the π^0 polarizabilities. In the work [10] the analysis of the angular distributions of pions from the process $\gamma\gamma \to \pi^0\pi^0$ in this energy region has resulted in $(\alpha_1+\beta_1) = 1.00 \pm 0.05$ (in units of $10^{-4}\,\mathrm{fm}^3$).

The fit of the data [11] for the total cross section of the process $\gamma\gamma \to \pi^0\pi^0$, using dispersion relations (DR) at fixed t (where t is the square of the total energy in $\gamma\gamma$ c.m.s.) with one subtraction for the invariant amplitudes [12] in the energy region from 270 up to 2000 MeV, has allowed for the determination of π^0 dipole polarizabilities $(\alpha_1+\beta_1)_{\pi^0} = 0.98 \pm 0.03$ and $(\alpha_1-\beta_1)_{\pi^0} = 1.6 \pm 2.2$. Here the σ meson was considered as an effective description of the strong S-wave $\pi\pi$ interaction using the broad Bright-Wigner resonance expression. The parameters of such a σ meson were found from the fit to the experimental data [11] in the energy region 270–825 MeV. As a result, a good description of the experimental data [11] was obtained for $\sqrt{t}$ =270–1700 MeV. However, this model predicts a strong rise in the total cross section at higher energies in contradiction with the experimental data.

In the present work we show that this discrepancy can be eliminated in the energy region at least up to 2.25 GeV by considering the quadrupole π^0 meson polarizabilities as free parameters.

An investigation of the process $\gamma\gamma \to \pi^0\pi^0$ at low and middle energies was also carried out in the framework of different theoretical models [13, 14, 15, 16]. However, these models did not allow a good description of the experimental data [11] on the total cross section in the full energy region from 270 to 2000 MeV. In the present work good agreement with the experimental data under consideration has been obtained in the full energy region from 270 to 2250 MeV.

The quadrupole polarizabilities of pions and nucleons were investigated in Ref. [17] where, in particular, the sum of the electric and magnetic polarizabilities of the pions and the sum and difference for the proton have been estimated for the first time using dispersion sum rules. In Ref. [18, 19] the quadrupole polarizabilities of the nucleons were calculated with help of dispersion relations and the results obtained were compared to predictions based upon chiral symmetry.

As was shown in Ref. [5, 20], the quadrupole polarizabilities give a big contribution to the cross section of Compton scattering on the π^0.

In the present paper, the contribution of the quadrupole polarizabilities of the π^0 meson to the total cross section of the process $\gamma\gamma \to \pi^0\pi^0$ is studied using dispersion relations at fixed t with a subtraction for invariant amplitudes. The subtraction functions are determined by the DRs in cross channels with two sub-

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tractions where the subtraction constants are connected with dipole and quadrupole pion polarizabilities. The subtractions in the DRs provide good convergence of the integrand expressions of these DRs and so increase the reliability of the calculations.

It is shown that the total cross section of the process $\gamma\gamma \to \pi^0\pi^0$ is very sensitive to the values of the quadrupole polarizabilities in the energy region higher than 1250 MeV. The fit of the experimental data [11, 21] to the process $\gamma\gamma \to \pi^0\pi^0$ using the DRs constructed has allowed for the determination of the values of the sum and the difference of the π^0 meson quadrupole polarizabilities for the first time.

In order to analyze these values of the quadrupole polarizabilities, the dispersion sum rules (DSR) for $(\alpha_2-\beta_2)$ and $(\alpha_2+\beta_2)$ are constructed. The prediction of DSRs and ChPT for the dipole polarizabilities of the charged and neutral pions are compared with the existing experimental values.

The paper is organized as follows. In Sec. II DRs for the invariant amplitudes of the process $\gamma\gamma \to \pi^0\pi^0$ are constructed. In Sec. III the dispersion sum rules for the dipole and quadrupole pion polarizabilities are constructed and analyzed. The determination of the π^0 quadrupole polarizabilities from the experimental data on the process $\gamma\gamma \to \pi^0\pi^0$ follows in Sec. IV. The main conclusions are presented in Sec. V. The details of the calculations of meson resonances are given in the Appendix.

II. DISPERSION RELATIONS FOR THE AMPLITUDES OF THE PROCESS $\gamma\gamma \to \pi^0\pi^0$

The dipole and quadrupole polarizabilities arise as $\mathcal{O}(\omega^2)$ and $\mathcal{O}(\omega^4)$ terms, respectively, in the expansion of the non-Born amplitude of Compton scattering over the initial photon energy ω. In terms of the electric α_l (l =1, 2) and magnetic β_l dipole and quadrupole polarizabilities, the corresponding effective interactions of $\mathcal{O}(\omega^2)$ and $\mathcal{O}(\omega^4)$ have the forms [18]:

$$H^{(2)}_{eff} = -\frac{1}{2}\,4\pi\,(\alpha_1\,\vec{E}^2 + \beta_1\,\vec{H}^2), \tag{1}$$

$$H^{(4)}_{eff} = -\frac{1}{12}4\pi\left(\alpha_2\,E^2_{ij} + \beta_2\,H^2_{ij}\right) \tag{2}$$

where

$$E_{ij} = \frac{1}{2}\,(\nabla_i\,E_j + \nabla_j\,E_i)\,,\; H_{ij} = \frac{1}{2}\,(\nabla_i\,H_j + \nabla_j\,H_i) \tag{3}$$

are the quadrupole strengths of the electric and magnetic fields.

The dipole polarizabilities (α_1 and β_1) of the pion measure the response of the pion to quasistatic electric and magnetic fields. On the other hand, the parameters α_2 and β_2 measure the electric and magnetic quadrupole moments induced in the pion in the presence of an applied field gradient.

In order to determine the quadrupole polarizabilities of π^0 meson, we will consider the process $\gamma\gamma \to \pi^0\pi^0$. This process is described by the following invariant variables

$$t = (k_1 + k_2), \quad s = (p_1 - k_1)^2, \quad u = (p_1 - k_2)^2 \tag{4}$$

where $p_1(p_2)$ and $k_1(k_2)$ are the pion and photon 4-momenta.

We will consider the helicity amplitudes M_{++} and M_{+-} [22] which are expressed through Prange's amplitudes [23] T_1 and T_2 as

$$\begin{aligned} M_{++} &= -\frac{1}{2t}\,(T_1 + T_2)\,, \\ M_{+-} &= -\frac{T_1 - T_2}{2[(s-\mu^2)^2 + st]}. \end{aligned} \tag{5}$$

These amplitudes have no kinematical singularities or zeros and define the cross section of the process $\gamma\gamma \to \pi^0\pi^0$ as follows

$$\begin{aligned} \frac{d\sigma_{\gamma\gamma\to\pi^0\pi^0}}{d\Omega} &= \frac{1}{256\pi^2}\sqrt{\frac{(t-4\mu^2)}{t^3}}\,\Big\{t^2|M_{++}|^2 \\ &\quad + \frac{1}{16}t^2(t-4\mu^2)^2\sin^4\theta^*|M_{+-}|^2\Big\} \end{aligned} \tag{6}$$

where θ^* is the angle between the photon and the pion in the c.m.s. of the process $\gamma\gamma \to \pi^0\pi^0$ and μ is the π^0 meson mass.

Constructing the DR at fixed t with one subtraction at $s = \mu^2$ for the amplitude M_{++} we have:

$$\begin{aligned} &ReM_{++}(s,t) = ReM_{++}(s=\mu^2,t) \\ &+\frac{(s-\mu^2)}{\pi}P\int\limits_{4\mu^2}^{\infty} ds'\;ImM_{++}(s',t)\left[\frac{1}{(s'-s)(s'-\mu^2)}\right. \\ &\left.-\frac{1}{(s'-u)(s'-\mu^2+t)}\right]. \end{aligned} \tag{7}$$

Via the crossing symmetry this DR is identical to a DR with two subtractions.

We determine the subtraction function $ReM_{++}(s=\mu^2,t)$ with the help of the DR at fixed $s=\mu^2$ with two subtractions using the crossing symmetry between the s and u channels

$$\begin{aligned} &ReM_{++}(s=\mu^2,t) = \\ &= M_{++}(s=\mu^2,0) + t\frac{dM_{++}(s=\mu^2,t)}{dt}\bigg|_{t=0} \\ &+\frac{t^2}{\pi}\left\{P\int\limits_{4\mu^2}^{\infty}\frac{ImM_{++}(t',s=\mu^2)\;dt'}{t'^2(t'-t)}\right. \\ &\left.+\int\limits_{4\mu^2}^{\infty}\frac{ImM_{++}(s',u=\mu^2)\;ds'}{(s'-\mu^2)^2(s'-\mu^2+t)}\right\}. \end{aligned} \tag{8}$$

Taking into account the expressions of the sum and the difference of the generalized electric and magnetic polarizabilities of any multipolar order through invariant amplitudes [24], we determine the subtraction constants $M_{++}(s=\mu^2,t=0)$ and $dM_{++}(s=\mu^2,t)/dt\big|_{t=0}$ in terms of differences of the dipole ($(\alpha_1-\beta_1)_{\pi^0}$) and quadrupole ($(\alpha_2-\beta_2)_{\pi^0}$) polarizabilities

$$M_{++}(s=\mu^2,t=0) = 2\pi\mu(\alpha_1-\beta_1)_{\pi^0},$$
$$\left.\frac{dM_{++}(s=\mu^2,t)}{dt}\right|_{t=0} = \frac{\pi\mu}{6}(\alpha_2-\beta_2)_{\pi^0}. \tag{9}$$

The DRs for the amplitude $M_{+-}(s,t)$ have the same expressions (7) and (8) but with the substitutions: $M_{++}\to M_{+-}$ and $ImM_{++}\to ImM_{+-}$. The subtraction constants are equal in this case to

$$M_{+-}(s=\mu^2,t=0) = \frac{2\pi}{\mu}(\alpha_1+\beta_1)_{\pi^0},$$
$$\left.\frac{dM_{+-}(s=\mu^2,t)}{dt}\right|_{t=0} = \frac{\pi}{6\mu}(\alpha_2+\beta_2)_{\pi^0}. \tag{10}$$

III. DISPERSION SUM RULES FOR THE PION POLARIZABILITIES

The DSR for the difference of the dipole polarizabilities was obtained in Ref. [25] using DR at fixed $u=\mu^2$ without subtractions for the amplitude M_{++}:

$$(\alpha_1-\beta_1) = \frac{1}{2\pi^2\mu}\left\{\int\limits_{4\mu^2}^{\infty}\frac{ImM_{++}(t',u=\mu^2)\;dt'}{t'} + \int\limits_{4\mu^2}^{\infty}\frac{ImM_{++}(s',u=\mu^2)\;ds'}{s'-\mu^2}\right\}. \tag{11}$$

The DSR for the sum of the dipole polarizabilities reads

$$(\alpha_1+\beta_1) = \frac{\mu}{\pi^2}\int\limits_{4\mu^2}^{\infty}\frac{ImM_{+-}(s',t=0)\;ds'}{s'-\mu^2} = \frac{1}{2\pi^2}\int\limits_{\frac{3}{2}\mu}^{\infty}\frac{\sigma_T(\nu)d\nu}{\nu^2} \tag{12}$$

where σ_T is the total cross section of the $\gamma\pi$ interaction and ν is the photon energy in the lab. system.

The DSRs for the difference and the sum of the quadrupole polarizabilities can be obtained with the help of the DRs at fixed $u=\mu^2$ with one subtraction for the amplitudes M_{++} and M_{+-}, respectively:

$$(\alpha_2-\beta_2) = \frac{6}{\pi^2\mu}\left\{\int\limits_{4\mu^2}^{\infty}\frac{ImM_{++}(t',u=\mu^2)\;dt'}{t'^2} - \int\limits_{4\mu^2}^{\infty}\frac{ImM_{++}(s',u=\mu^2)\;ds'}{(s'-\mu^2)^2}\right\}, \tag{13}$$

$$(\alpha_2+\beta_2) = \frac{6\mu}{\pi^2}\left\{\int\limits_{4\mu^2}^{\infty}\frac{ImM_{+-}(t',u=\mu^2)\;dt'}{t'^2} - \int\limits_{4\mu^2}^{\infty}\frac{ImM_{+-}(s',u=\mu^2)\;ds'}{(s'-\mu^2)^2}\right\}. \tag{14}$$

The DSRs for the charged pions are saturated by the contributions of the $\rho(770)$, $b_1(1235)$, $a_1(1270)$, and $a_2(1320)$ mesons in the s channel and σ, $f_0(980)$, $f_0^1(1370)$, $f_2(1270)$, and $f_2^1(1525)$ in the t channel. For the π^0 meson the contribution from the ρ, $\omega(782)$, and $\phi(1020)$ mesons are considered in the s channel and from the same mesons as for the charged pions in the t channel.

The parameters of the ρ, ω, ϕ, b_1, a_2, f_2, and f_2^1 mesons are given by the Particle Data Group [26]. The parameters of the $f_0(980)$, $f_0^1(1370)$, and a_1 mesons are taken as follows:
$f_0(980)$: $m_{f_0}=980$ MeV [26], $\Gamma_{f_0}=70$ MeV (the average of the PDG [26] estimate), $\Gamma_{f_0\to\gamma\gamma}=0.39\times10^{-3}$ MeV [26], $\Gamma_{f_0\to\pi\pi}=0.84\,\Gamma_{f_0}$ [27];
$f_0^1(1370)$: $m_{f_0^1}=1434$ MeV [28], $\Gamma_{f_0^1}=173$ MeV [28], $\Gamma_{f_0^1\to\gamma\gamma}=0.54\times10^{-5}$ MeV [29], $\Gamma_{f_0^1\to\pi\pi}=0.26\,\Gamma_{f_0^1}$ [30];
$a_1(1270)$: $m_{a_1}=1230$ MeV [26], Γ_{a_1} =425 MeV (the average value of the PDG estimate [26]), $\Gamma_{a_1\to\gamma\pi^\pm}=0.64$ MeV [31].

For the σ meson we use the values of mass and decay widths found in Ref. [12]: $m_\sigma=547$ MeV, $\Gamma_\sigma=1204$ MeV, $\Gamma_{\sigma\to\gamma\gamma}=0.62$ keV.

The results of the calculations of the DSRs for the dipole (in units of $10^{-4}\,\text{fm}^3$) and quadrupole (in units of $10^{-4}\,\text{fm}^5$) polarizabilities are presented in Table I for the charged and in Table II for the neutral pions. The contributions of the $f_2(1525)$ meson to the DSRs for $(\alpha_2+\beta_2)_{\pi^\pm}$ and for $(\alpha_2+\beta_2)_{\pi^0}$ were found to be very small (~ 0.0004) and were not included in the tables. The errors indicated are due to uncertainties in the parameters of the mesons considered.

The influence of the integration limit in the DSRs (11)–(14) on results of the calculations was investigated. The analysis has shown that the integrand expressions for the quadrupole polarizabilities converge very quickly and the integration up to 2 GeV gives practically 100%. For the dipole polarizabilities the integrand expressions converge more slowly, particularly for their difference. While for the sums of the dipole polarizabilities, an integration limit of 5 GeV gives about 99%, the integration in the DSRs for the differences up to 10 GeV introduces uncertainties of $\sim 1\%$ for $(\alpha_1-\beta_1)_{\pi^\pm}$ and $\sim 5\%$ for $(\alpha_1-\beta_1)_{\pi^0}$. In the present work we performed the integrations up to 10 GeV for the quadrupole polarizabilities and the sum

of the dipole polarizabilities and up to 100 GeV for the differences of the dipole polarizabilities.

The investigation within the framework of ChPT in a two loop analysis $\mathcal{O}(p^6)$ [32] has yielded the following values for the dipole polarizabilities of the charged pions:

$$(\alpha_1 - \beta_1)_{\pi^\pm} = 4.4 \pm 1.0, \tag{15}$$
$$(\alpha_1 + \beta_1)_{\pi^\pm} = 0.3 \pm 0.1. \tag{16}$$

For the π^0 meson ChPT has predicted [14]

$$(\alpha_1 - \beta_1)_{\pi^0} = -1.90 \pm 0.20, \tag{17}$$
$$(\alpha_1 + \beta_1)_{\pi^0} = 1.15 \pm 0.30. \tag{18}$$

A recent experiment at the Mainz Microtron MAMI [6] has resulted in

$$(\alpha_1 - \beta_1)_{\pi^\pm} = 11.6 \pm 1.5_{stat} \pm 3.0_{syst} \pm 0.5_{mod}. \tag{19}$$

This value of $(\alpha_1 - \beta_1)_{\pi^\pm}$ is close to the result of Ref. [1].

The dipole polarizabilities of the π^0 meson were determined by investigating the process $\gamma\gamma \to \pi^0\pi^0$ in Ref. [9, 10, 12]. These works have given: $(\alpha_1 + \beta_1)_{\pi^0} = 1.00 \pm 0.05$, $(\alpha_1 - \beta_1)_{\pi^0} = -0.6 \pm 1.8$ [9, 10] and $(\alpha_1 + \beta_1)_{\pi^0} = 0.98 \pm 0.03$, $(\alpha_1 - \beta_1)_{\pi^0} = -1.6 \pm 2.2$ [12].

As seen from Table I, the DSR results in $(\alpha_1 - \beta_1)_{\pi^\pm} = 13.60 \pm 2.15$. This value of $(\alpha_1 - \beta_1)_{\pi^\pm}$ is in agreement within the errors with the experimental data (19) but differs significantly from the ChPT prediction (15). On the other hand, the DSR calculations for π^0 meson dipole polarizabilities (Table II) are not in conflict within the errors with both experimental data of Ref. [10, 12] and the ChPT predictions (17).

IV. DETERMINATION OF THE π^0 MESON QUADRUPOLE POLARIZABILITIES

To determine the quadrupole polarizabilities of the π^0 meson, we fit experimental data on the process $\gamma\gamma \to \pi^0\pi^0$ in the region $\sqrt{t} = 270$–2250 MeV. As the data of Ref. [11] have large errors in the energy region 1600–2000 MeV and so cannot be correctly used to determine the cross section behavior at higher energies, we in addition consider the data of Ref. [21] in the region of 2000–2250 MeV. We fit these experimental data using the DRs (7)–(8) for the amplitude M_{++} and the corresponding DRs for M_{+-} where the difference and the sum of the quadrupole polarizabilities are free parameters. The values of the dipole polarizabilities $(\alpha_1 - \beta_1)_{\pi^0}$ and $(\alpha_1 + \beta_1)_{\pi^0}$ and the parameters of the σ meson are taken from Ref. [12]. In order to improve the description of the $f_2(1270)$ meson resonance peak, the effective radius (r_f) and the decay width $\Gamma_{f_2 \to \gamma\gamma}$ of the meson are considered as free parameters, too.

As a result, the following values have been found:

$$(\alpha_2 + \beta_2)_{\pi^0} = (-0.181 \pm 0.004) \times 10^{-4}\,\text{fm}^5, \tag{20}$$

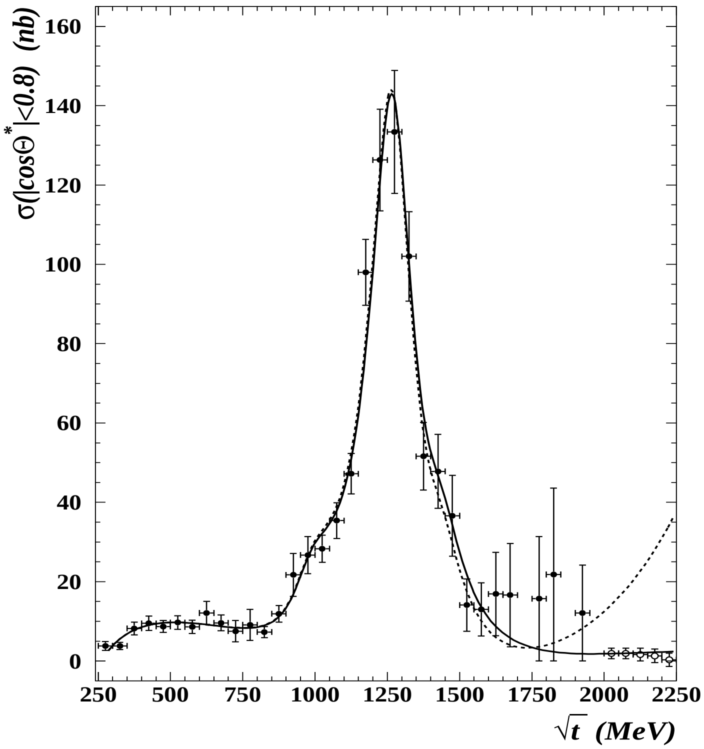

FIG. 1: The total cross section of the reaction $\gamma\gamma \to \pi^0\pi^0$. The solid curve is the result of the fit. The dashed curve corresponds to the quadrupole polarizabilities calculated with the help of the DSRs. The full circles are data from Ref. [11] and the open ones are data from Ref. [21].

$$(\alpha_2 - \beta_2)_{\pi^0} = 39.70 \pm 0.02 \times 10^{-4}\,\text{fm}^5, \tag{21}$$

and

$$r_f = 0.96 \pm 0.01\ \text{fm}, \quad \Gamma_{f_2 \to \gamma\gamma} = 3.05 \pm 0.11\ \text{keV}. \tag{22}$$

Note that the value of r_f determined in Ref. [33] is equal to 1.05 ± 0.24 fm. The value of $\Gamma_{f_2 \to \gamma\gamma}$ practically coincides with the one found in [11] and differs from the value presented by the Particle Data Group [26] (2.61 ± 0.24 keV).

Fig. 1 shows a fit to the total cross section of the process $\gamma\gamma \to \pi^0\pi^0$ in the energy region up to $\sqrt{t} = 2.25$ GeV [11, 21] using the values of $(\alpha_2 + \beta_2)_{\pi^0}$, $(\alpha_2 - \beta_2)_{\pi^0}$, r_f, and $\Gamma_{f_2 \to \gamma\gamma}$ found (the solid curve). The dashed curve corresponds to the values of the quadrupole polarizabilities obtained with the help of the DSRs (13) and (14) using the meson parameters described in Sec. III. As the predictions of the DSR (13) for $(\alpha_2 - \beta_2)_{\pi^0}$ effectively coincide with the values (21) found in this analysis, the difference between the predictions and the experiment in the region of 1750–2250 MeV is caused by the deviation of $\sim 5\%$ of the DSR result for $(\alpha_2 + \beta_2)_{\pi^0}$ from the experimental value.

The sensitivity of the calculations of the cross section of the process $\gamma\gamma \to \pi^0\pi^0$ to values of the difference and the sum of the quadrupole polarizabilities is shown in Fig. 2. The solid and dashed curves in this figure present results

TABLE I: The DSR predictions for the polarizabilities of the charged pions in units of $10^{-4}\,\mathrm{fm}^3$ for the dipole polarizabilities and $10^{-4}\,\mathrm{fm}^5$ for the quadrupole polarizabilities.

	ρ	b_1	a_1	a_2	f_0	$f_0^{\prime}$	σ	f_2	Σ	$\Delta\Sigma$
$(\alpha_1-\beta_1)$	-1.15	0.93	2.26	1.51	0.58	0.02	9.45	-	13.60	2.15
$(\alpha_1+\beta_1)$	0.063	0.021	0.051	0.031	-	-	-	-	0.166	0.024
$(\alpha_2-\beta_2)$	0.78	-0.25	-0.63	-0.41	0.31	0.01	25.94	-	25.75	7.03
$(\alpha_2+\beta_2)$	-0.027	-0.003	-0.011	0.013	-	-	-	0.149	0.121	0.064

TABLE II: The DSR predictions for the polarizabilities of the π^0 meson.

	ρ	ω	ϕ	f_0	$f_0^{\prime}$	σ	f_2	Σ	$\Delta\Sigma$
$(\alpha_1-\beta_1)$	-1.58	-12.56	-0.04	0.60	0.02	10.07	-	-3.49	2.13
$(\alpha_1+\beta_1)$	0.080	0.721	0.001	-	-	-	-	0.802	0.035
$(\alpha_2-\beta_2)$	1.06	9.53	0.02	0.32	0.01	28.78	-	39.72	8.01
$(\alpha_2+\beta_2)$	-0.035	-0.284	0	-	-	-	0.148	-0.171	0.067

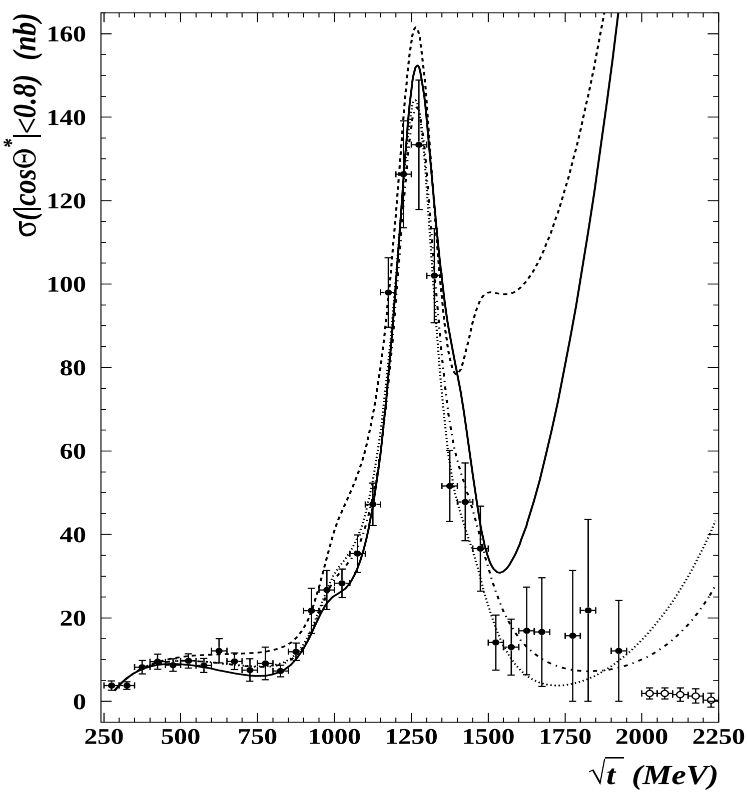

FIG. 2: The sensitivity of the cross section calculations to different values of the quadrupole polarizabilities. The solid (dashed) curve corresponds to $(\alpha_2-\beta_2)_{\pi^0}$ bigger (less) by 5% than the experimental value. The dotted (dashed-dotted) curve presents the result with the same 5% deviation for $(\alpha_2+\beta_2)_{\pi^0}$.

of the calculations when the value of $(\alpha_2-\beta_2)_{\pi^0}$ is bigger and less by 5% than the experimental value (21). The dotted and dashed-dotted curves correspond to the same 5% deviations from the value (20) for $(\alpha_2+\beta_2)_{\pi^0}$.

As seen from this figure, the result of the calculations of the cross section is very sensitive to the value of $(\alpha_2-\beta_2)_{\pi^0}$ in the overall energy region under consideration particularly at $\sqrt{t} > 1400$ MeV. On the other hand, 5% changes of $(\alpha_2+\beta_2)_{\pi^0}$ do not influence the result of the calculations in the energy region up to 1400 MeV but lead to an essential difference from the experimental data at the higher energies.

All of this indicates a high sensitivity of our calculations to the values of the quadrupole π^0 meson polarizabilities, particularly in the energy region above 1400 MeV.

The values of $(\alpha_2-\beta_2)_{\pi^0}$ and $(\alpha_2+\beta_2)_{\pi^0}$ we found are consistent within the errors with the predictions of the DSRs (13) and (14).

V. CONCLUSIONS

The DRs at fixed t with one subtraction at $s=\mu^2$ have been constructed for the invariant amplitudes of the process $\gamma\gamma\to\pi^0\pi^0$. The subtraction functions were determined with the help of the DRs at fixed $s=\mu^2$ with two subtractions at $t=0$, where the subtraction constants were expressed through the dipole and quadrupole polarizabilities. These DRs, where the sum and the difference of the quadrupole π^0 polarizabilities were free parameters, were used to fit the experimental data [11, 21] for the total cross sections of the process $\gamma\gamma\to\pi^0\pi^0$ in the energy region from 270 to 2250 MeV. As a result the values of the sum and the difference of the quadrupole polarizabilities have been found for the first time:

$$(\alpha_2+\beta_2)_{\pi^0} = (-0.181\pm 0.004)\times 10^{-4}\,\mathrm{fm}^5,$$

$$(\alpha_2-\beta_2)_{\pi^0} = 39.70\pm 0.02\times 10^{-4}\,\mathrm{fm}^5.$$

In addition, this fit allowed us to determine the values of the effective radius of the $f_2(1270)$ meson and its decay width into two photons: $r_f = 0.96\pm 0.01$ fm, $\Gamma_{f_2\to\gamma\gamma} = 3.05\pm 0.11$ keV.

To analyze the results obtained, the DSRs for the difference and sum of the quadrupole polarizabilities have been constructed.

The values of $(\alpha_2-\beta_2)_{\pi^0}$ and $(\alpha_2+\beta_2)_{\pi^0}$ extracted from the experimental data on the process $\gamma\gamma\to\pi^0\pi^0$ are in good agreement with the calculations of the DSRs.

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APPENDIX

The contributions of the vector and axial-vector mesons (ρ, ω, ϕ, a_1, and b_1) are calculated with the help of the expression

$$\begin{aligned} ImM_{++}^{(V)}(s,t) &= \mp s\; ImM_{+-}^{(V)}(s,t) \\ &= \mp 4g_V^2 s\frac{\Gamma_0}{(m_V^2-s)^2+\Gamma_0^2} \end{aligned} \quad (A.1)$$

where m_V is the meson mass, the sign "+" corresponds to the contribution of the a_1 and b_1 mesons and

$$g_V^2 = 6\pi\sqrt{\frac{m_V^2}{s}}\left(\frac{m_V}{m_V^2-\mu^2}\right)^3\Gamma_{V\to\gamma\pi},$$

$$\Gamma_0 = \left(\frac{s-4\mu^2}{m_V^2-4\mu^2}\right)^{\frac{3}{2}} m_V\Gamma_V. \quad (A.2)$$

Here Γ_V and $\Gamma_{V\to\gamma\pi}$ are the full width and the decay width into $\gamma\pi$ of these mesons, respectively.

The contributions of the a_2 meson are calculated using a narrow width approximation.

$$\begin{aligned} ImM_{++}^{(a_2)}(s,t) &= -\frac{1}{2}g_a^2\pi\left[s^2-s(4\mu^2-2t)+\mu^4\right. \\ &\left.+\frac{s(s+\mu^2)^2}{2m_a^2}\right]\delta(s-m_a^2), \end{aligned} \quad (A.3)$$

$$ImM_{+-}^{(a_2)}(s,t) = -g_a^2\pi\left[\mu^2-t-\frac{(s+\mu^2)^2}{4m_a^2}\right]\delta(s-m_a^2) \quad (A.4)$$

where

$$g_a^2 = 160\pi\left(\frac{m_a}{m_a^2-\mu^2}\right)^5\Gamma_{a_2\to\gamma\pi^\pm}.$$

For calculating the contribution of the σ, $f_0(980)$, $f_0^1(1370)$, and f_2 mesons we use the following expressions:

$$\begin{aligned} ImM_{++}^{\sigma}(t,s) &= \frac{g_\sigma\Gamma_{0\sigma}}{(m_\sigma^2-t)^2+\Gamma_{0\sigma}^2}, \\ ImM_{++}^{f_0}(t,s) &= \frac{g_{f_0}\Gamma_{0f_0}}{(m_{f_0}^2-t)^2+\Gamma_{0f_0}^2}, \\ ImM_{+-}^{f_2}(t,s) &= \frac{g_{f_2}\Gamma_{0f_2}}{(m_{f_2}^2-t)^2+\Gamma_{0f_2}^2} \end{aligned} \quad (A.5)$$

where

$$\begin{aligned} g_\sigma &= 8\pi\frac{m_\sigma+\sqrt{t}}{\sqrt{t}}\left(\frac{\frac{2}{3}\Gamma_{\sigma\to\pi\pi}\Gamma_{\sigma\to\gamma\gamma}}{m_\sigma\sqrt{m_\sigma^2-4\mu^2}}\right)^{\frac{1}{2}}, \\ \Gamma_{0\sigma} &= \frac{\Gamma_\sigma}{2}(\sqrt{t}+m_\sigma)\left(\frac{t-4\mu^2}{m_\sigma^2-4\mu^2}\right)^{\frac{1}{2}}, \end{aligned} \quad (A.6)$$

$$\begin{aligned} g_{f_0} &= 16\pi\left(\frac{\frac{2}{3}\Gamma_{f_0\to\pi\pi}\Gamma_{f_0\to\gamma\gamma}}{m_{f_0}\sqrt{m_{f_0}^2-4\mu^2}}\right)^{\frac{1}{2}}, \\ \Gamma_{0f_0} &= \Gamma_{f_0}m_{f_0}\left(\frac{t-4\mu^2}{m_{f_0}^2-4\mu^2}\right)^{\frac{1}{2}}, \end{aligned} \quad (A.7)$$

$$\begin{aligned} g_{f_2} &= 160\pi\frac{m_{f_2}^{3/2}}{t(m_{f_2}^2-4\mu^2)^{\frac{5}{4}}}\sqrt{\frac{D_2(m_{f_2}^2)}{D_2(t)}\;\Gamma_{f_2\to\pi\pi}\Gamma_{f_2\to\gamma\gamma}}, \\ \Gamma_{0f_2} &= \Gamma_{f_2}\frac{m_{f_2}^2}{\sqrt{t}}\left(\frac{t-4\mu^2}{m_{f_2}^2-4\mu^2}\right)^{\frac{5}{2}}\frac{D_2(m_{f_2}^2)}{D_2(t)}. \end{aligned} \quad (A.8)$$

The decay form factor D_2 is given according to Ref. [11]

$$D_2(t) = 9+3(q\,r_f)^2+(q\,r_f)^4, \quad q^2=\frac{1}{4}(t-4\mu^2) \quad (A.9)$$

where r_f is the effective interaction radius of the f_2 meson. The factor $(m_\sigma+\sqrt{t})$ in the relations for g_σ and $\Gamma_{0\sigma}$ is introduced to get a more correct expression for a broad Breit-Wigner resonance.

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